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Rescattering in DIS and other QCD processes

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Abstract. I discuss two issues related to interactions of partons moving in a color field. In deep inelastic scattering reinteractions of the struck quark in the color field of the target may neutralize the overall color exchange from the target. This allows the target to emerge with a rapidity gap to the rest of the produced hadrons: Diffractive DIS. In hard hadron-hadron interactions similar rescattering takes place, and again allows rapidity gap formation. Because of the different color environment in lepton *vs.* hadron induced processes the rescattering probabilities are not the same, hence the diffractive parton distributions are process-dependent. For both lepton and hadron induced diffraction the hard scattering subprocesses and their logarithmic scaling violations are, however, the same as in inclusive scattering.
Another type of rescattering occurs when quarks and gluons move in the QCD vacuum condensate. Their long distance propagation is likely to be strongly affected by the condensate, which may explain why usual perturbation fails for soft processes. We have modelled the condensate by a 'vacuum' gluon field which carries zero four-momentum and couples to quarks and gluons in a gauge invariant way. The dressing of the free propagators by the vacuum gluons can be calculated exactly and shows that colored partons have a finite life-time. This framework may help address issues of analyticity and unitarity in theories where the fundamental fields differ from the asymptotic states of the S-matrix.

PACS. 12.38.-t Quantum chromodynamics – 13.60.-r Photon and charged-lepton interactions with hadrons – 13.85.-t Hadron-induced high- and super-high-energy interactions

1 Hard diffraction from parton rescattering

1.1 Diffractive deep inelastic scattering (DDIS)

In the intuitive picture of inclusive Deep Inelastic Scattering (DIS), $eN \to eX$, a color string-field is formed between the struck parton and the target remnant covering the whole rapidity interval between them. The breaking of the string during the hadronization process fills the rapidity interval with hadrons; the probability of a rapidity gap would be expected to decrease exponentially with the gap size. However, HERA measurements [3] show that about 10% of DIS final states have a large gap. The DDIS to DIS cross section ratio depends weakly on the virtuality Q^2 of the photon at fixed Bjorken x_B, implying that DDIS is a Bjorken-scaling leading-twist process. The large photon virtuality suggests that it should be possible to study hard diffraction using the analysis tools of QCD perturbation theory. Collins [4] has indeed shown that diffractive processes induced by virtual photons such as DDIS do factorize as a product of diffractive parton distributions times the usual hard parton cross sections. This diffractive factorization theorem does not, however, extend to hard diffractive hadron–hadron scattering. The fraction of events with a large rapidity gap has, indeed, been found to be only $\sim 1\%$ in hadron induced diffraction [5], an order of magnitude below the DDIS rate.

I shall discuss [1] how the dynamics of DDIS, as well as diffraction in hadron induced processes, can arise from the soft rescattering in deep inelastic scattering discussed by Brodsky, Hoyer, Marchal, Peigné and Sannino (BHMPS) [6]. As it turns out, the soft rescattering dynamics is in several respects similar to the phenomenologically successful Soft Color Interaction (SCI) model previously proposed by Edin, Ingelman and Rathsman [7].

In the conventional parton model picture of DIS the virtual photon is absorbed on a quark in the target. The struck quark then propagates through the target with (nearly) the velocity of light and may interact with the target spectators via longitudinal (A^+) gluon exchange. This soft rescattering is described (in a general gauge) by the path-ordered exponential (Wilson line) in the expression for the parton distributions given by the QCD factorization theorem. If the photon momentum is chosen to be in the negative z-direction the rescatterings occur (in the Bjorken limit) at an instant of Light-Front (LF) time $x^+ = t+z$. Hence the rescattering can formally be included in the definition of the $x^+ = 0$ target LF wave function,

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even though the exchanges occur a finite ordinary time $t \simeq -z$ *after* the hard virtual photon interaction.

In the SCI model diffraction arises from soft gluon exchanges between the target spectators and the diffractive (projectile) system which leaves the target in a color-singlet state. The color currents induced by the hard virtual photon interaction must therefore be screened before the onset of hadronization. This is achieved by parton rescattering, which in the BHMPS approach occurs via essentially instantaneous gluon exchange analogous to 'Coulomb' scattering. The rescatterings involve on-shell intermediate states which at small x_B provide the imaginary phase associated with diffractive scattering (pomeron exchange). The rescattering is part of the standard leading-twist DIS dynamics and thus is not power-suppressed at large Q^2. It also causes shadowing effects in nuclear targets [6] and Bjorken-scaling single-spin asymmetries in DIS [8].

1.2 Parton distributions and rescattering

According to the QCD factorization theorem, which is based on the properties of perturbative diagrams at arbitrary orders, the quark distribution of the nucleon is given by the matrix element

$$f_{q/N}(x_B, Q^2) = \frac{1}{8\pi} \int dx^- \exp(-i x_B p^+ x^-/2) \times \langle N(p)|\bar{\psi}(x^-)\gamma^+ W[x^-;0]\,\psi(0)|N(p)\rangle \qquad (1)$$

where all fields are evaluated at equal LF time $x^+ = 0$ and small transverse separation $x_\perp \sim 1/Q$. The Wilson line $W[x^-;0]$,

$$W[x^-;0] = \mathrm{P}\exp\left[ig\int_0^{x^-} dw^- A^+(w^-)\right] \qquad (2)$$

physically represents rescattering of the struck quark on the target spectators. Only the longitudinal (A^+) component appears in the path ordered exponential (2). This component has no x^+ derivative in the Lagrangian and is therefore "instantaneous" in x^+. Soft transverse ($A^\perp$) gluon exchange does not occur within the coherence length of the virtual photon, $x^- \sim 1/m_p x_B = 2\nu/Q^2$ determined by the Fourier transform of (1) in the target rest frame, and later interactions do not affect the DIS cross section. This ensures that the DIS cross section is proportional to the nucleon matrix element (1); however, as shown in [6] the presence of the Wilson line precludes a probabilistic interpretation of the parton distributions.

The Wilson line reduces to unity in LF gauge, $A^+ = 0$. Hence it is sometimes assumed that the path-ordered exponential is just a gauge artifact; *i.e.*, that the A^+ gluon exchanges do not affect the DIS cross section at leading twist. This would conflict with our conventional understanding of diffraction and shadowing as arising from the interference of amplitudes with dynamical phases.

This question was studied in some detail in the perturbative model of BHMPS [6]. The contribution to the inclusive DIS cross section from the struck quark rescattering indeed vanishes in LF gauge, consistent with the Wilson line reducing to unity. This follows from the form of the LF gluon propagator,

$$d_{LF}^{\mu\nu}(k) = \frac{i}{k^2 + i\varepsilon}\left[-g^{\mu\nu} + \frac{n^\mu k^\nu + k^\mu n^\nu}{k^+}\right] \qquad (3)$$

where $n^2 = 0$ and $n \cdot A = A^+$. The second term is a gauge artifact which cannot contribute to physical amplitudes. In particular, it was seen that the poles at $k^+ = 0$ generated by the propagator (3) are absent from the full BHMPS amplitudes, although they contribute to individual Feynman diagrams. The contributions of the individual diagrams also depend on the $i\epsilon$ prescription used at $k^+ = 0$, but the sum of all diagrams is prescription independent.

The BHMPS diagrams with struck quark rescattering vanish individually in the prescription $k^+ \to k^+ - i\epsilon$ due to a cancellation between the two terms in the square brackets of the LF propagator (3). However, the remaining $k^+ = 0$ poles in diagrams involving interactions within the target spectator system (such as between p_2 and p' in Fig. 1 below) then give a non-vanishing contribution to the scattering amplitude. In fact their contribution must be equal to that of the struck quark rescattering in Feynman gauge (the first term in (3)), as required by gauge invariance. LF gauge is thus subtle in that *interactions between spectators contribute to the DIS cross section* through the gauge dependent $k^+ = 0$ pole terms.

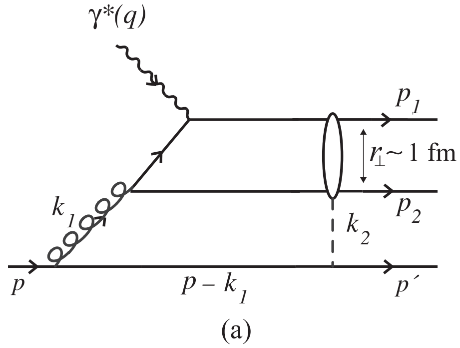
1.3 Mechanism for diffraction

The perturbative model of DIS studied in [6] provides insights into the dynamics of diffractive DIS and allows one to see why the *hard subprocess* is the same as in inclusive DIS, as required by the diffractive factorization theorem [4]. Requiring a rapidity gap between the target and diffractive system imposes a condition only on the *soft rescattering* of the struck quark, namely that the target system emerges as a color singlet. As we shall see, this will not modify the Q^2-dependence of the cross section.

I refer to [6] for a detailed discussion of the properties of the DDIS model amplitudes shown in Fig. 1. Here I only give a qualitative picture of the dynamics of $ep \to e'Xp'$ as suggested by perturbation theory:

(i) A gluon (k_1) which carries a small fraction $k_1^+/p^+ \sim x_{\mathbb{P}}$ of the proton momentum splits into (a) a $q\bar{q}$ or (b) a gg pair. This is a soft process within the target dynamics, consequently the parton pair has a large transverse size ~ 1 fm.

(ii) The virtual photon is absorbed on (a) one of the quarks in the pair, or (b) scatters via $\gamma^* g \to Q\bar{Q}$ to a compact, $r_\perp \sim 1/Q$ quark pair. The struck quark (or $Q\bar{Q}$ pair) carries the asymptotically large photon momentum, $p_1^- \simeq 2\nu$. The parton (p_2) that did not interact with the photon also has large $p_2^- \simeq (m_q^2 + p_{2\perp}^2)/p_2^+$ owing to its small $p_2^+ \sim x_B p^+$.



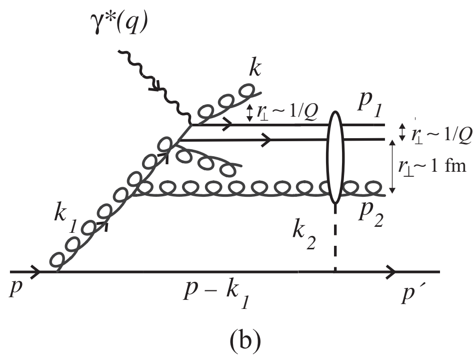


Fig. 1. Low-order rescattering correction to DIS in the parton model frame where the virtual photon momentum is along the negative z-axis with $q = (q^+, q^-, \boldsymbol{q}_\perp) \simeq (-m_p x_B, 2\nu, \mathbf{0})$ and the target is at rest, $p = (m_p, m_p, \mathbf{0})$. The struck parton absorbs nearly all the photon momentum, $p_1 \simeq (0, 2\nu, \boldsymbol{p}_{1\perp})$ (aligned jet configuration). In **a** the virtual photon strikes a quark and the diffractive system is formed by the $q\bar{q}$ pair (p_1, p_2) which rescatters coherently from the target via 'instantaneous' longitudinal (A^+) gluon exchange with momentum k_2. In **b** the $Q\bar{Q}$ quark pair which is produced in the $\gamma^* g \to Q\bar{Q}$ subprocess has a small transverse size $r_\perp \sim 1/Q$ and rescatters like a gluon. The diffractive system is then formed by the $(Q\bar{Q})\,g$ system. The possibility of hard gluon emission close to the photon vertex is indicated. Such radiation (labeled k) emerges at a short transverse distance from the struck parton and is not resolved in the rescattering

(iii) Multiple soft longitudinal gluon exchange (labeled k_2) turns the color octet $q\bar{q}$ of Fig. 1a or the $(Q\bar{Q})g$ of Fig. 1b into a color singlet diffractive system. (The compact $Q\bar{Q}$ pair behaves as a high energy gluon since its internal structure is not resolved during the soft rescattering.)

The rescattering which turns the diffractive system into a color singlet occurs within the target, before it has time to hadronize. The color currents of the gluon exchanges are thus shielded before a color string can form between the target and the diffractive system, hence no hadrons are produced in the rapidity interval $\sim \log(1/x_B)$ between them.

The effective scattering energy of the diffractive system on the target spectator is given by $p_2^- \propto 1/x_B$. As required by analyticity the crossing-even two-gluon exchange amplitude of Fig. 1 is imaginary at low x_B, implying that the intermediate state between the two gluon exchanges is on-shell. Rescattering is necessary to generate the dominantly imaginary amplitude expected for diffraction.

1.4 Higher order effects at the hard vertex

In the above discussion I considered the hard virtual photon vertex only at lowest order. Just as in inclusive DIS, hard gluon emission and virtual loops give rise to a scale dependence in the parton distributions, and to corrections of higher order in α_s to the subprocess cross section. In the parton model frame of Fig. 1b (where the target proton is at rest) perturbative gluons radiated at the hard vertex have $k_\perp \gg p_{2\perp}$ and $k^+ \lesssim p_2^+$. Hence their rapidities $\sim \log(k^-/k_\perp) \sim \log(k_\perp/k^+) \gtrsim \log(p_{2\perp}/p_2^+)$ tend to be larger than the rapidity of the lower edge of the diffractive system, given by p_2. The hadrons resulting from the hard gluon radiation therefore do not populate the rapidity gap. The gluons are radiated at a short transverse distance from the struck parton and their transverse velocity $v_\perp \sim k_\perp/k^- \sim k^+/k_\perp$ is small. The struck parton and its radiated gluons thus form a transversally compact system whose internal structure is not resolved in the soft rescattering.

According to the above discussion, the size of the rapidity gap and the soft rescattering are unaffected by higher order corrections at the virtual photon vertex. This is corroborated by the SCI Monte Carlo, where one observes only small variations of the Δy_{max} distribution when varying the parton shower cut-off and thereby the amount of perturbative radiation. Thus the Q^2-dependence of the diffractive parton distributions and the subprocess amplitudes are the same as in inclusive DIS, in accordance with the diffractive factorization theorem [4].

1.5 Diffraction in hard hadron collisions

Our description of diffraction in deep inelastic lepton scattering can be extended to hard diffractive hadronic collisions. As required by dimensional scaling, only a single parton from the projectile and target participate in the hard subprocess. These leading twist subprocesses (including their higher order corrections) are the same for inclusive and diffractive scattering. The soft rescattering of the hard partons and their spectators is constrained by the requirement of a rapidity gap in the final state. The partonic systems on either side of the gap must be color singlets in order to prevent the formation of a color string in the later hadronization phase.

The soft rescattering is quite different in hadron collisions as compared to DIS. In hadron collisions both the projectile and target spectator systems are colored. The rescattering gluons (k_2 in Fig. 1) can thus couple also to the projectile remnant. In Fig. 2 the compact $q\bar{q}$ pair, which is created in the hard gluon–gluon collision, is not resolved by the soft rescattering and therefore retains it color. Together with the projectile remnant it forms a

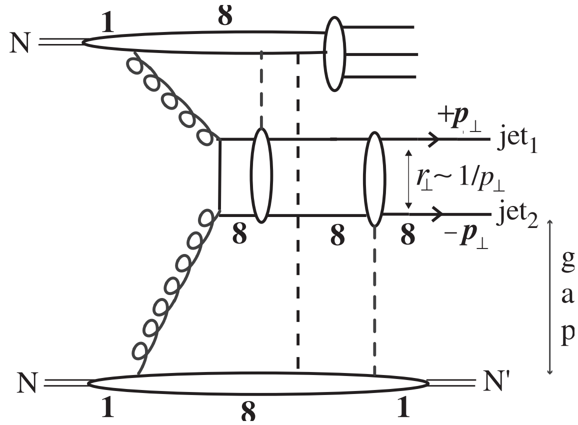


Fig. 2. Illustration of diffraction through rescattering in $NN \to 2\ jets + X$ in analogy with the DIS case in Fig. 1. The compact $\mathrm{q\bar{q}}$ pair which forms the jets is assumed to be in a color octet (**8**) configuration. This pair rescatters coherently and thus retains its color

transversally extended color octet dipole which can rescatter softly from the target remnant. A rapidity gap is formed between the target remnant and the compact $\mathrm{q\bar{q}}$ pair if the target remnant emerges as a color singlet after the rescattering. The probability for this is, however, different from target neutralization in DIS. This is observed experimentally: the ratio of diffractive to inclusive cross sections is of the order of $\sim 1\%$ for a variety of hard processes observed at the Tevatron as compared to the $\sim 10\%$ ratio of DDIS/DIS. In the small-x region, these ratios are approximately independent of the momentum fractions in the proton. The data is well accounted for by the SCI model [9].

1.6 Summary of hard diffraction

Hard diffractive processes such as diffractive DIS provide new insight into the dynamics of QCD. I emphasized that the subprocesses with large momentum transfer are the same in all inclusive reactions: they involve a *single constituent* from the projectile and target and are given by perturbative QCD. The parton distributions reflect the LF wave functions of the colliding particles *and* the soft rescattering of the partons emerging from the hard subprocess. The rescattering is mediated by longitudinal gluons and occurs 'instantaneously' in LF time as the partons pass the spectators. Hard partons which are radiated in the subprocess itself are not resolved by the soft longitudinal gluons which scatter coherently off the color charge of the struck parton.

In a diffractive process the soft rescattering is constrained by the requirement that the diffractive systems on either side of the rapidity gap are color singlets. Since the configurations of color-charged spectators are different in virtual photon and the various hadron induced diffractive processes, this requirement means that diffractive parton distributions are process dependent. Comparisons of the parton distributions for different projectiles and rapidity gap configurations can thus give valuable information on the rescattering dynamics.

This description of hard diffractive reactions provides predictions at several levels of accuracy:

1. The Q^2 dependence of all diffractive parton distributions is the same as that of inclusive parton distributions. For DDIS this is a statement of the diffractive factorization theorem [4].
2. The dependence on the fractional momentum x carried by the parton is similar for diffractive and inclusive distributions, assuming that the momentum transferred in the rescattering is small.
3. The dependence on the diffractive mass of the diffractive parton distributions arises from the underlying (non-perturbative) $g \to \mathrm{q\bar{q}}$ and $g \to gg$ splittings in the case of quark and gluon distributions, respectively.

2 QCD Green functions in a gluon field

2.1 Coupling of quarks and gluons to a vacuum field

The presence of a gluon and quark 'condensate' in the QCD ground state is a plausible reason for the observed long distance properties of QCD [10]. The condensate apparently prevents quarks and gluons from propagating over long distances, while acting as a superfluid for color singlet hadrons. I shall here briefly describe an effort [2] to model the gluon condensate effects by coupling quarks and gluons to a 'vacuum' gluon field Φ which carries vanishing momentum in a covariant gauge. We integrate over the Lorentz and color components of Φ^a_μ with a gaussian weight. This ensures Lorentz and gauge invariance and introduces a dimensionful parameter Λ which characterizes the magnitude of the vacuum field. In effect, we modify QCD perturbation theory (PQCD) by expanding around non-vanishing gluon field configurations.

We work in a standard covariant gauge and define the coupling of quarks to the vacuum gluon field Φ by

$$\mathcal{L}_{\Phi \mathrm{q}} = -g\bar{\psi}\Phi^\mu\gamma_\mu\psi \tag{4}$$

This interaction is invariant under the gauge transformation $\psi \to U(x)\psi$ with $U(x) \in \mathrm{SU}(N)$ provided Φ transforms as

$$\Phi^\mu \to U(x)\Phi^\mu U(x)^\dagger \tag{5}$$

For gluons we use

$$\mathcal{L}_{\Phi \mathrm{g}} = -\mathrm{Tr}\left[F^{\mu\nu}F^\Phi_{\mu\nu}\right] \tag{6}$$

where

$$F^\Phi_{\mu\nu} = \partial_\mu\Phi_\nu - \partial_\nu\Phi_\mu + ig([\Phi_\mu, A_\nu] - [\Phi_\nu, A_\mu]) \tag{7}$$

transforms as $F^\Phi_{\mu\nu} \to U(x)F^\Phi_{\mu\nu}U(x)^\dagger$.

As the field Φ is meant to describe the long wavelength (vacuum condensate) effects we take it to carry zero momentum, i.e., to be independent of the coordinate x. To preserve Lorentz and gauge invariance we average over all components of Φ_μ^a with a gaussian weight,

$$\int_{-\infty}^{\infty} \prod_{\mu,a} d\Phi_\mu^a \exp\left[\frac{1}{2\Lambda^2}\Phi_\nu^b \Phi_b^\nu\right] \tag{8}$$

where Λ is a parameter with the dimension of mass. In a perturbative expansion we may interpret (8) as giving a Φ 'propagator'

$$iD_{\Phi,\mu\nu}^{ab}(p) = -\Lambda^2 g_{\mu\nu}\delta^{ab}(2\pi)^4\delta^4(p) \tag{9}$$

2.2 Dressed quark and gluon propagators

In the limit of a large number of colors, $N \to \infty$ with g^2N fixed [11], we find explicit expressions for quark and gluon propagators, dressed to all orders in the couplings (4) and (6) of the zero-momentum vacuum field Φ. The dependence on the mass parameter Λ in (8) then enters via

$$\mu^2 = g^2 N \Lambda^2 \tag{10}$$

The dressed quark propagator $S(p)$ is most easily found by noting that it satisfies the implicit equation shown in Fig. 3,

$$\begin{aligned} iS(p) &= \frac{i}{\not{p}} + C_F(-ig)^2(-\Lambda^2)\frac{i}{\not{p}}\gamma^\mu iS(p)\gamma_\mu iS(p) \\ \Rightarrow \quad & \not{p}S(p) = 1 - \tfrac{1}{2}\mu^2\,\gamma^\mu S(p)\gamma_\mu S(p) \end{aligned} \tag{11}$$

For a bare quark mass $m = 0$ we find two solutions,

$$S_1(p) = \frac{2\not{p}}{p^2 + \sqrt{p^2(p^2-4\mu^2)}} \tag{12}$$

$$S_2(p) = -\frac{1}{\mu^2}\left(\not{p} \pm \sqrt{p^2+\mu^2/2}\right) \tag{13}$$

of which the latter implies spontaneous chiral symmetry breaking. The coupling to the vacuum field has replaced the 'on-shell' $p^2 = 0$ pole of the bare propagator with a branch cut. This implies that the dressed quark decays in time. After Fourier transforming $p^0 \to t$ we find explicitly

$$|S_1(t,\boldsymbol{p})| \underset{|t|\to\infty}{\sim} \mathcal{O}\left(1/\sqrt{|t|}\right) \tag{14}$$

Hence quarks cannot appear as asymptotic states in an S-matrix.

The gluon propagator has a double color line structure, both of which are dressed by planar Φ 'loops'. No closed implicit equation like that for quarks (Fig. 3) can be written down. However, since each Φ loop contributes the same factor $\propto \mu^2/p^2$ the dressing may be evaluated by counting the number of planar diagrams. The result for the dressed gluon propagator in Landau gauge is

$$iD_{ab}^{\mu\nu}(p) = \frac{i}{4\mu^2}\,P_T^{\mu\nu}(p)\,\left[1 - {}_2F_1\left(-\frac{1}{2},\frac{1}{2},2,-\frac{32\mu^2}{p^2}\right)\right]\delta_{ab} \tag{15}$$

where $P_T^{\mu\nu}(p) = g^{\mu\nu} - p^\mu p^\nu/p^2$ and ${}_2F_1$ is a hypergeometric function. Surprisingly, the dressed gluon has a cut for spacelike momenta, $-32\mu^2 \le p^2 \le 0$. For $p^2 \to \infty$ the dressed propagator differs from the bare one by terms of $\mathcal{O}\left(\mu^2/p^2\right)$, i.e., the dressing gives only higher twist effects in hard processes. For $p^2 \to 0$ the propagator (15) is $\propto 1/\sqrt{p^2}$, and thus behaves as the quark propagator (12).

2.3 Quark–photon vertex

The dressed $\gamma q\bar{q}$ vertex $\Gamma^\mu(k,\bar{k})$ satisfies (at large N) the implicit equation of Fig. 4,

$$\Gamma^\mu(k,\bar{k}) = \gamma^\mu - \tfrac{1}{2}\mu^2\gamma^\rho S(k)\Gamma^\mu(k,\bar{k})S(\bar{k})\gamma_\rho \tag{16}$$

where $\bar{k} = k - p$ and p is the photon momentum. When the vertex is expanded on its independent Dirac components (16) reduces to a set of linear equations for the components, with a unique solution when the quark propagator S is given. Using the chiral symmetry conserving quark propagator S_1 of (12) we find

$$\begin{aligned} \Gamma^\mu(k,\bar{k}) = \frac{1}{1+2fk\cdot\bar{k}+f^2k^2\bar{k}^2}\Big\{&(1+fk\cdot\bar{k})\gamma^\mu \\ &-fi\gamma_5\,\epsilon^{\mu\nu\rho\sigma}\gamma_\nu k_\rho \bar{k}_\sigma + \frac{2f^2}{1-f^2k^2\bar{k}^2}(k^\mu \not{k}\bar{k}^2 + \bar{k}^\mu \not{\bar{k}} k^2) \\ &+\frac{f(1+f^2k^2\bar{k}^2)}{1-f^2k^2\bar{k}^2}(k^\mu \not{\bar{k}} + \bar{k}^\mu \not{k})\Big\} \end{aligned} \tag{17}$$

where $f \equiv \mu^2 a_k a_{\bar{k}}$ with $a_p \equiv (1-\sqrt{1-4\mu^2/p^2})/2\mu^2$. Multiplying (16) by $p_\mu = (k-\bar{k})_\mu$ and rewriting (11) as

$$\tfrac{1}{2}\mu^2\gamma^\mu S(k)\gamma_\mu = S(k)^{-1} - \not{k} \tag{18}$$

it is readily verified that any solution of (16) respects the Ward-Takahashi identity

$$p_\mu \Gamma^\mu(k,\bar{k}) = S(k)^{-1} - S(\bar{k})^{-1} \tag{19}$$

Fig. 3. Implicit equation satisfied by the dressed quark propagator $S(p)$. The *dashed line* denotes the Φ 'propagator' (9). In the limit of a large number of colors only planar Φ 'loops' contribute

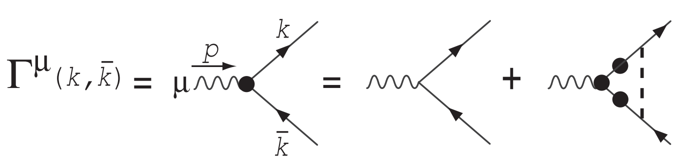


Fig. 4. Implicit equation for the quark-photon vertex $\Gamma^\mu(k,\bar{k})$

2.4 Photon self-energy

The dressed quark loop correction to the photon propagator is given by the dressed quark propagator and quark-photon vertex as indicated in Fig. 5. In terms of the solutions of the DS equations for the quark propagator (11) and vertex (16) the photon self-energy correction $\Pi^{\mu\nu}(p)$ is

$$\begin{aligned}\Pi^{\mu\nu}(p) &= ie^2 N \int \frac{d^dk}{(2\pi)^d} \mathrm{Tr}\left[\gamma^\nu S(k) \Gamma^\mu(k,\bar{k}) S(\bar{k})\right] \\ &= \frac{2}{d-2}\frac{ie^2N}{\mu^2}\int\frac{d^dk}{(2\pi)^d}\left\{\mathrm{Tr}\left[\Gamma^\mu(k,\bar{k})\gamma^\nu\right] - 4g^{\mu\nu}\right\} \qquad (20)\end{aligned}$$

where we used the relation (16) for the vertex and $\gamma^\nu = \gamma_\rho\gamma^\nu\gamma^\rho/(2-d)$. Multiplying by p_μ and using the Ward-Takahashi identity (19) we find $p_\mu \Pi^{\mu\nu}(p) = 0$. Hence the self-energy is transverse,

$$\Pi^{\mu\nu}(p) = \Pi(p^2)\left(p^2 g^{\mu\nu} - p^\mu p^\nu\right) \qquad (21)$$

The loop integral expression (20) for $\Pi(p^2) = \Pi^\mu_{\ \mu}(p)/[(d-1)p^2]$ is fairly complicated, given the form (17) of $\Gamma^\mu(k,\bar{k})$. It is however, easy to check that the integral is UV convergent except for the term of $\mathcal{O}\left(\mu^0\right)$, which gives the standard short-distance PQCD behavior. The integral is *regular* in the IR limit $k \to 0$ (and $k \to p$).

It is interesting to note that the dressing affects the quark loop correction to the photon propagator even for spacelike photon momenta $p^2 < 0$. One would naively expect that the Φ field, which has infinite wavelength, would decouple from $q\bar{q}$ pairs of size $\sim 1/\sqrt{-p^2}$. It is in fact not difficult to give a formal proof of this decoupling [2] at each order μ^{2n}. However, it turns out that the loop integrals for $n \geq 2$ are infrared divergent in the case of massless ($m = 0$) quarks. With $2n$ zero-momentum gluons attached to a quark loop there are contributions of the form

$$A_n \sim \int d^4k\, k^{2n} \frac{k \cdot \bar{k}}{(k^2+m^2)^{2n+1}} \qquad (22)$$

For $m \neq 0$ the integral is IR regular, corresponding to a maximal size $\sim 1/m$ of the $q\bar{q}$ fluctuation. For $m = 0$, on the other hand, the dipole factors contributed by the Φ couplings favor large size quark pair contributions to the extent that the integral is singular at $k = 0$.

The dressing involves a coherent sum over all orders μ^{2n} and gives a finite result. Intuitively, this may be interpreted as a confinement effect: due to the finite propagation length of the dressed quark the maximal pair size is $\sim 1/\mu$.

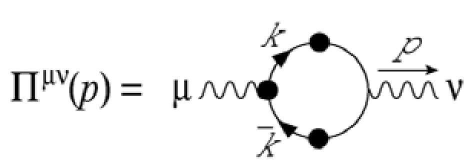


Fig. 5. Photon self-energy $\Pi^{\mu\nu}(p)$ dressed by the Φ field. The dressed propagators and vertex are indicated by a solid circle

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